### THE UNIVERSITY OF BRITISH COLUMBIA

# DEPARTMENT OF STATISTICS

TECHNICAL REPORT #261 (Correction to: TR #253)

# **RIZVI-SOBEL SUBSET SELECTION**

### WITH UNEQUAL SAMPLE SIZES

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#### CORRECTION TO: RIZVI-SOBEL SUBSET SELECTION WITH UNEQUAL SAMPLE SIZES Technical Report #253 Department of statistics, University of British Columbia by Constance van Eeden

The present manuscript contains a corrected version of Section 3 of the Technical Report #253.

#### The procedure for the smallest quantile

As in Section 2, the conditions on the  $F_i$  are the ones used by RS, i.e. the  $F_i$  satisfy

$$\max_{1 \le i \le k} F_{[i]}(y) = F_{[1]}(y) \text{ for all } y.$$
(3.1)

Further, as before, the  $F_i$  are continuous and have a unique  $\alpha$ -quantile, but in this case the integers  $r_i$  and  $c_i$  satisfy

$$1 \le r_i \le (n_i + 1)\alpha < r_i + 1 \le n_i + 1 \text{ and } 0 \le c_i \le n_i - r_i.$$
(3.2)

The proposed procedure is then

$$R_2$$
: put  $F_i$  in the subset  $\Leftrightarrow Y_{r_i,i} \le \min_{1 \le j \le k, j \ne i} Y_{r_j+c_j,j}$  (3.3)

and when  $F_i = F_{[1]}$  the probability of a correct selection is

$$P_{i,d_i}(CS \mid R_2) = P(Y_{r_i,i} \le \min_{1 \le j \le k, j \ne i} Y_{r_j + c_j,j}), \qquad (3.4)$$

where, as for the case of the largest quantile, the probability of correct selection when  $F_i = F_{[1]}$  depends on the *c*'s only through  $d_i = (c_1, \ldots, c_{i-1}, c_{i+1}, \ldots, c_k)$ .

Using (2.1) and (2.5) then gives, for the case where  $F_i = F_{[1]}$ ,

$$\left. \begin{array}{l}
P_{i,d_{i}}(CS \mid R_{2}) = \\
P(Y_{r_{i},i} \leq \min_{1 \leq j \leq k, j \neq i} Y_{r_{j}+c_{j}}) = \\
\int_{-\infty}^{\infty} \prod_{j \neq i} \left( 1 - I_{F_{j}(y)}(r_{j}+c_{j}, n_{j}-(r_{j}+c_{j}+1)) dI_{F_{i}(y)}(r_{i}, n_{i}-r_{i}+1) \right) \\
\geq \int_{-\infty}^{\infty} \prod_{j \neq i} \left( 1 - I_{u}(r_{j}+c_{j}, n_{j}-(r_{j}+c_{j})+1) dI_{u}(r_{i}, n_{i}-r_{i}+1). \right) \end{array} \right\} (3.6)$$

Calling this lowerbound on  $P_{i,d_i}(CS \mid R_2)$ ,  $L^*_{i,d_i}(CS \mid R_2)$ , we have, because  $0 \le c_i \le n_i - r_i$  and  $I_u(r, n - r + 1)$  is for fixed  $u \in (0, 1)$  decreasing in r,

$$A_i^* \le L_{i,d_i}^*(CS \mid R_2) \le B_i^*, i = 1, \dots, k$$
(3.7)

where, for  $i = 1, \ldots, k$ ,

$$A_i^* = \int_0^1 \prod_{j \neq i} \left( 1 - I_u(r_j, n_j - r_j + 1) \right) dI_u(r_i, n_i - r_i + 1)$$
(3.8)

and

$$B_i^* = \int_0^1 \prod_{j \neq i} \left( 1 - I_u(n_j, 1) \right) dI_u(r_i, n_i - r_i + 1).$$
(3.9)

Further note that (by a proof similar to the one for (2.15))

$$\sum_{i=1}^{k} A_i^* = 1 \text{ for all } r_i, n_i \text{ satisfying (3.2)},$$
(3.10)

implying that  $\min_{1 \le i \le k} A_i^* \le 1/k$  and by a proof similar to the one for (4.1)

$$B_i^* = 1 - \frac{n_i!(n_j + r_i - 1)!}{(r_i - 1)!(n_i + n_j)!} \ i \neq j$$

when k = 2.

Finally, Theorem 2.1 with, for i = 1, ..., k,  $(A_i, B_i)$  replaced by  $(A_i^*, B_i^*)$ , gives  $(c_1, ..., c_k)$  and the possible  $P^*$  such that  $\min_{1 \le i \le k} P_{i,d_i}(CS \mid R_2) \ge P^*$ .